

# Accuracy, Precision and Uncertainties

# Precision

- How close your results are to each other
- How repeatable they are
- The number of decimal places or significant figures that an instrument can measure

# Accuracy

- How close your results are to the true value of what you are measuring

# Example 1

- The actual temperature of a classroom is  $18.5^{\circ}\text{C}$
- A digital thermometer measures it to be  $12.231^{\circ}\text{C}$ 
  - is this accurate or precise?

It is very precise because it gives a value to 5 significant figures, but it is not very accurate because it is a long way from the true temperature

## Example 2

- The actual temperature of a classroom is  $18.5^{\circ}\text{C}$
- A digital thermometer measures it to be  $19^{\circ}\text{C}$ 
  - is this accurate or precise?

It is accurate because it gives a value close to the actual value, but it is not very precise because it only reads to two significant figures

# Random errors

- Human errors e.g. reaction time when using a stopwatch or parallax errors when reading a graduated scale
- Unpredictable environmental changes e.g. wind blowing during an outdoor experiment to measure the speed of sound
- Random errors can be reduced by repeating the experiment and averaging your results
- Random errors mainly affect precision

# Systematic errors

- Equipment errors e.g. digital scales being incorrectly zeroed or a thermometer that consistently reads 3°C too high
- Systematic errors can be reduced by replacing equipment, calibrating equipment or using multiple sets of equipment to compare readings
- Systematic errors mainly affect accuracy

# Summary questions 1

Please write your answers in your books and attempt these questions without looking back at the previous slides. Spend 5 minutes on this.

1. Describe the difference between accuracy and precision
2. Describe the difference between random and systematic errors and give an example of each

# Summary questions 1 - answers

Who would like to share their answers with the class?

# Percentage difference

- The percentage difference is a simple calculation to compare the value obtained in an experiment to its true value
- It gives an idea of how accurate your result is

$$\% \text{ difference} = \frac{|\text{experimental value} - \text{true value}|}{\text{true value}} \times 100 \%$$

- The vertical bars mean take the 'modulus' (absolute value) – so you ignore the minus sign if the numerator is negative
- A rule of thumb is that if your % difference is less than 10% then your experimental result is fairly accurate

# Example 1

- An experiment to measure the time taken to travel a certain distance gives a reading of 11.8 s. The true time taken is 10.3 s. Calculate the percentage difference and comment on the accuracy of the experiment.

$$\% \text{ difference} = \frac{|11.8 - 10.3|}{10.3} \times 100 \% = 14.6 \%$$

It is not a very accurate experiment because the % difference is greater than 10 %.

## Example 2

- An experiment to measure the density of steel gives a value of  $7895 \text{ kgm}^{-3}$ . The true value is  $8050 \text{ kgm}^{-3}$ . Calculate the percentage difference and comment on the accuracy of the experiment.

$$\% \text{ difference} = \frac{|7895 - 8050|}{8050} \times 100 \% = 1.9 \%$$

It is a very accurate experiment because the % difference is much less than 10 %.

# Summary questions 2

Please write your answers in your book and attempt these questions without looking back at the previous slides. Spend 5 minutes on this.

1. An experiment to measure the temperature of an aluminium block gives a result of  $94.2^{\circ}\text{C}$ . The true value is  $81.8^{\circ}\text{C}$ . Calculate the percentage difference and comment on the accuracy of the experiment.
2. An experiment to measure the mass of a copper block gives a result of  $1.01\text{ kg}$ . The true value is  $1.06\text{ kg}$ . Calculate the percentage difference and comment on the accuracy of the experiment.

# Summary questions 2 - answers

## Question 1

$$\% \text{ difference} = \frac{|94.2 - 81.8|}{81.8} \times 100 \% = 15.2 \%$$

It is not a very accurate experiment because the % difference is greater than 10 %.

# Summary questions 2 - answers

## Question 2

$$\% \text{ difference} = \frac{|1.01 - 1.06|}{1.06} \times 100 \% = 4.7 \%$$

It is an accurate experiment because the % difference is less than 10 %.

# Absolute uncertainties

- These are expressed as  $\pm$  the smallest increment that your instrument or measuring device can measure.
- For example a stopwatch that can measure to 0.1 s has an absolute uncertainty of  $\pm 0.1$  s.
- So, if this stopwatch gives a reading of 5.0 s it should really be quoted as  $5.0 \text{ s} \pm 0.1 \text{ s}$ .
- This means that the range of true values is 4.9 s to 5.1 s.

# Percentage uncertainties

- These are the absolute uncertainties expressed as a percentage of the measured value
- So, for the stopwatch that produces a reading of  $5.0 \text{ s} \pm 0.1 \text{ s}$ , the percentage uncertainty is:

$$\frac{0.1}{5.0} \times 100 = 2 \%$$

# Absolute and Percentage uncertainties

- It is important to note that the absolute uncertainty is fixed as it is directly related to the resolution (how small the measuring increments are) of the measuring device.
- Percentage uncertainty can be reduced however, by increasing the range of your measured value.

# Absolute and Percentage uncertainties

- As seen previously, if the stopwatch is used to measure 5 s, the percentage uncertainty is:

$$\frac{0.1}{5.0} \times 100 = 2 \%$$

- If the same stopwatch is used to measure 50 s, the percentage uncertainty is:

$$\frac{0.1}{50} \times 100 = 0.2 \%$$

# Absolute and Percentage uncertainties

- This is why in an experiment you want your measured range to be as large as possible – because it reduces the % uncertainty in your result.
- For example measuring the period of a pendulum oscillation by timing over multiple swings rather than just one swing

# Absolute and Percentage uncertainties

- At A Level, you need to be able to calculate the percentage uncertainty of a quantity derived from a formula - for example, the percentage uncertainty in speed from measured values of distance and time.
- To work out the percentage uncertainty of a quantity derived from a formula, the uncertainties from the measured values **MUST BE EXPRESSED AS PERCENTAGE UNCERTAINTIES FIRST** – this is usually the first step in these questions.
- If the measured quantities in the formula are being multiplied or divided **YOU ALWAYS ADD THE PERCENTAGE UNCERTAINTIES.**
- If you have a squared (or cubed) quantity then you **ADD** the uncertainties just the same as if it were any other two quantities being multiplied together – **TAKE CARE HERE, MANY A LEVEL PUPILS LOSE MARKS WITH THIS.**

# Example 1

A runner runs in a race. The distance is measured as  $250 \text{ m} \pm 1 \text{ m}$  and the time is measured as  $31 \text{ s} \pm 0.1 \text{ s}$ . Calculate the percentage uncertainty in their average speed.

**Step 1: Work out the percentage uncertainties of distance and time**

$$\text{Distance} = 1/250 \times 100 = 0.40 \%$$

$$\text{Time} = 0.1/31 \times 100 = 0.32 \%$$

**Step 2: Look at the formula for average speed: average speed = distance/time**

Distance and time are divided – this means that to calculate the % uncertainty in speed, you ADD the % uncertainties in distance and time.

**Step 3: Percentage uncertainty in average speed =  $0.40 + 0.32 = \underline{0.72\%}$**

## Example 2

A car's mass is measured as  $1200 \text{ kg} \pm 25 \text{ kg}$  and its velocity is measured as  $18 \text{ m/s} \pm 1 \text{ m/s}$ . Calculate the percentage uncertainty in the car's momentum.

**Step 1: Work out the percentage uncertainties of mass and velocity**

$$\text{Mass} = 25/1200 \times 100 = 2.08 \%$$

$$\text{Velocity} = 1/18 \times 100 = 5.56 \%$$

**Step 2: Look at the formula for momentum:      momentum = mass x velocity**

Mass and velocity are multiplied – this means that to calculate the % uncertainty in momentum, you ADD the % uncertainties in mass and velocity.

**Step 3: Percentage uncertainty in momentum =  $2.08 + 5.56 = \underline{7.64\%}$**

# Example 3

The length of a cube's edge measures  $40 \text{ cm} \pm 2 \text{ cm}$ . Its mass measures  $5 \text{ kg} \pm 0.15 \text{ kg}$ . Calculate the percentage uncertainty in its density.

Step 1: Work out the percentage uncertainties of length and mass

$$\text{Length} = 2/40 \times 100 = 5.0 \%$$

$$\text{Mass} = 0.15/5 \times 100 = 3.0 \%$$

Step 2: Look at the formula for density:  $\text{density} = \text{mass} / \text{volume}$

Step 3: Look at the formula for volume:  $\text{volume} = \text{length}^3$

Mass and volume are divided – this means that to calculate the % uncertainty in density, you ADD the % uncertainties in mass and volume. To calculate the % uncertainty in volume, you need to ADD the % uncertainties in length THREE TIMES BECAUSE IT IS CUBED.

Step 4: Percentage uncertainty in density =  $3.0 + 5.0 + 5.0 + 5.0 = \underline{18.0 \%}$

# Summary questions 3

Please write your answers in your book and attempt these questions without looking back at the previous slides. Spend 10 minutes on this.

1. A cyclist travels  $3500 \text{ m} \pm 60 \text{ m}$  in a time of  $1200 \text{ s} \pm 30 \text{ s}$ . Calculate the percentage uncertainty in their average speed.
2. An object with a mass of  $80 \text{ kg} \pm 2 \text{ kg}$  travels with a velocity of  $250 \text{ m/s} \pm 5 \text{ m/s}$ . Calculate the percentage uncertainty in its kinetic energy.

# Summary questions 3 - answers

## Question 1

A cyclist travels a distance of  $3500 \text{ m} \pm 60 \text{ m}$  in a time of  $1200 \text{ s} \pm 30 \text{ s}$ . Calculate the percentage uncertainty in their average speed.

Step 1: Work out the percentage uncertainties of distance and time

$$\text{Distance} = 60/3500 \times 100 = 1.7 \%$$

$$\text{Time} = 30/1200 \times 100 = 2.5 \%$$

Step 2: Look at the formula for average speed: average speed = distance/time

Step 3: Percentage uncertainty in average speed =  $1.7 + 2.5 = \underline{4.2 \%}$

# Summary questions 3 - answers

## Question 2

An object with a mass of  $80 \text{ kg} \pm 2 \text{ kg}$  travels with a velocity of  $250 \text{ m/s} \pm 5 \text{ m/s}$ . Calculate the percentage uncertainty in its kinetic energy.

Step 1: Work out the percentage uncertainties of mass and velocity

$$\text{Mass} = \frac{2}{80} \times 100 = 2.5 \%$$

$$\text{Velocity} = \frac{5}{250} \times 100 = 2.0 \%$$

Step 2: Look at the formula for kinetic energy:  $\text{kinetic energy} = 0.5 \times \text{mass} \times \text{velocity}^2$

Step 3: Percentage uncertainty in kinetic energy =  $2.5 + 2.0 + 2.0 = \underline{6.5 \%}$

The end